ECON0106 Tutorial 6 **2026-2027**

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Feedback for Test 2

Exercise 4 (Section 7)

(i)

Proof. For each θ, the probability of generating a sequence of flips $x_1, ..., x_m$ is

$$\theta^{\sum_{i=1}^m x_i}(1-\theta)^{m-\sum_{i=1}^m x_i}$$

We have

$$f_m(\theta) := f(\theta \mid x_1, ..., x_m) = \frac{f(\theta)\left(\theta^{\sum_{i=1}^m x_i}(1-\theta)^{m-\sum_{i=1}^m x_i}\right)}{\int f(\theta)\left(\theta^{\sum_{i=1}^m x_i}(1-\theta)^{m-\sum_{i=1}^m x_i}\right)d\theta}$$

$$f_n(\theta) := f(\theta \mid x'_1, ..., x'_n) = \frac{f(\theta)\left(\theta^{\sum_{i=1}^n x'_i}(1-\theta)^{n-\sum_{i=1}^n x'_i}\right)}{\int f(\theta)\left(\theta^{\sum_{i=1}^n x'_i}(1-\theta)^{n-\sum_{i=1}^n x'_i}\right)d\theta}$$

Denote

$$C := \frac{\int f(\theta)\left(\theta^{\sum_{i=1}^m x_i}(1-\theta)^{m-\sum_{i=1}^m x_i}\right)d\theta}{\int f(\theta)\left(\theta^{\sum_{i=1}^n x'_i}(1-\theta)^{n-\sum_{i=1}^n x'_i}\right)d\theta}$$

There is

$$\frac{f_m(\theta)}{f_n(\theta)} = \frac{\theta^{\sum_{i=1}^m x_i - \sum_{i=1}^n x'_i}}{(1-\theta)^{n-m+\sum_{i=1}^m x_i - \sum_{i=1}^n x'_i}} \cdot C$$

Since $\sum_{i=1}^m x_i \geq \sum_{i=1}^n x'_i$ and $n \geq m$, $\frac{f_m(\theta)}{f_n(\theta)}$ is non-decreasing in θ. As we have established $F(\theta \mid x_1, ..., x_m) \geq_{MLR} F(\theta \mid x'_1, ..., x'_n)$, by proposition 2, we have $\mathbb{E}_F[\theta \mid x_1, ..., x_m] \geq \mathbb{E}_F[\theta \mid x'_1, ..., x'_n]$. □

(ii)

Proof.

$$\frac{f(\theta \mid x_1, ..., x_m)}{g(\theta \mid x_1, ..., x_m)} = \frac{f(\theta)\left(\theta^{\sum_{i=1}^m x_i}(1-\theta)^{m-\sum_{i=1}^m x_i}\right)}{g(\theta)\left(\theta^{\sum_{i=1}^m x_i}(1-\theta)^{m-\sum_{i=1}^m x_i}\right)} \cdot \underbrace{\frac{\int g(\theta)\left(\theta^{\sum_{i=1}^m x_i}(1-\theta)^{m-\sum_{i=1}^m x_i}\right)d\theta}{\int f(\theta)\left(\theta^{\sum_{i=1}^m x_i}(1-\theta)^{m-\sum_{i=1}^m x_i}\right)d\theta}}_{D}$$

$$= \frac{f(\theta)}{g(\theta)} \cdot D$$

If $f \geq_{MLR} g$, then $\frac{f(\theta|x_1,...,x_m)}{g(\theta|x_1,...,x_m)}$ is non-decreasing. Hence,

$$\mathbb{E}_F[\theta \mid x_1, ..., x_m] \geq \mathbb{E}_G[\theta \mid x_1, ..., x_m].$$

□

Exercise 8 (Section 7)

1.

Proof. Notice that

$$\begin{aligned}
\mathbb{E}[u(Y)] &= \mathbb{E}[u(X+Z)] \\
&= \mathbb{E}[\mathbb{E}[u(X+Z) \mid X]] \\
&\leq \mathbb{E}[u(\mathbb{E}[X+Z \mid X])] \\
&= \mathbb{E}[u(X)]
\end{aligned}$$

where the inequality is derived from Jensen's inequality. Notice that the converse is true as $F \geq_{SOSD} G$ does not imply $\mathbb{E}_F = \mathbb{E}_G$. □

2.

Proof. Since $F \geq_{SOSD} G$ means that $\mathbb{E}_F[u(x)] \geq \mathbb{E}_G[u(x)]$ for any $u(\cdot)$ concave. Take $u(x) = x$ which is linear and we have the result. □

3.

Proof. Equal Expectation is by definition. For Variance, we have

$$\begin{aligned}
\mathbb{V}[Y] &= \mathbb{V}[X+Z] \\
&= \mathbb{V}[X] + \mathbb{V}[Z] \\
&\geq \mathbb{V}[X]
\end{aligned}$$

where the second equality is by zero covariance between X and Z. □

4.

Proof. $F \geq_{FOSD} G$ means that $\mathbb{E}_F[u(x)] \geq \mathbb{E}_F[u(x)]$ for any u non-decreasing. Trivially, this implies that this is true for all non-decreasing concave u. Immediately, the converse is not true. □